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Diffusion effects in the oscillation of vapor–gas bubbles in a sound field

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Abstract

The gas–vapor bubbles oscillating in the acoustic field are considered. The diffusion of gas and vapor components inside the bubble is taken into account. The analytical formula is obtained for the amplitude of the phase-transition rate. On the base of this formula the influence of the initial concentration of the gas was investigated. It is shown that the presence of relatively small amount of inert gas in the bubble can considerably decrease the phase-transition rate. This is related with the mutual diffusion of the components and so called "screen" effect of the inert gas at the bubble surface.

This is because the vapor component loses its ability to penetrate rapidly through the shielding gas layer on the surface of the bubble.

The effect of the kinetics of phase-transition rate (accommodation coefficient) on the intensity of phase-transition rate was also studied.

1. Introduction

Mathematical model for spherically symmetric process around vapor–gas bubbles has been formulated in [1,2], and their small oscillations have been investigated in detail in [3–9]. The dynamics of vapor–gas bubbles has important bearing, in particular, on sound propagation in the top of the ocean. The description of the case in this situation is far more complicated than in the case of a gas bubble or a vapor bubble.

1.1. Fundamental equations

The system of equations describing radially symmetric oscillations of a bubble filled with a liquid vapor and a gas that is insoluble in the liquid is given in [7], where it is assumed that a uniform pressure exists in the bubble and interdiffusion of the components of the vapor–gas mixture is taken into account.

The equations of energy, continuity, and the state of the phases in spherical Eulerian coordinates (r, t) have the form [7]:

$$\rho_{\mathrm{v}}\frac{\mathrm{d}u_{\mathrm{v}}}{\mathrm{d}t}+\rho_{\mathrm{g}}\frac{\mathrm{d}u_{\mathrm{g}}}{\mathrm{d}t}=\frac{p}{\rho}\frac{\mathrm{d}p}{\mathrm{d}t}+\frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda\frac{r^2\partial T}{\partial r}\right)+\rho D\frac{\partial k}{\partial r}\frac{\partial(u_{\mathrm{v}}-u_{\mathrm{g}})}{\partial r}, \tag{1}$$

$$\frac{\partial\rho_{\mathrm{v}}}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}[\rho_{\mathrm{v}}r^2(v+w_{\mathrm{v}})]=0 \quad (0\leqslant r\leqslant R(t)), \tag{2}$$

$$\frac{\partial\rho_{\mathrm{g}}}{\partial t}+\frac{1}{r^2}\frac{\partial}{\partial r}[\rho_{\mathrm{g}}r^2(v+w_{\mathrm{g}})]=0, \tag{3}$$

$$\rho_{\mathrm{g}}w_{\mathrm{g}}=-\rho_{\mathrm{v}}w_{\mathrm{v}}=\rho D\partial k/\partial r, \tag{4}$$

$$p=p_{\mathrm{v}}+p_{\mathrm{g}}=(\rho_{\mathrm{g}}B_{\mathrm{g}}+\rho_{\mathrm{v}}B_{\mathrm{v}})T=\rho BT, \quad T_{\mathrm{v}}=T_{\mathrm{g}}=T, \tag{5}$$

$$u_{\mathrm{v}}=c_{\mathrm{vv}}T, \quad u_{\mathrm{g}}=c_{\mathrm{vg}}T, \tag{6}$$

$$\rho_\ell\left(\frac{\partial u_\ell}{\partial t}+v_\ell\frac{\partial u_\ell}{\partial r}\right)=\frac{1}{r^2}\frac{\partial}{\partial r}\left(\lambda_\ell r^2\frac{\partial T_\ell}{\partial r}\right)+12\mu_{\mathrm{e}}\frac{v_{\mathrm{e}}^2}{r^2}(R(t)<r<\infty), \tag{7}$$

$$r^2v_\ell=R^2v_{\ell\sigma} \quad u_\ell=c_\ell T_\ell \quad \rho_\ell=\text{const}, \tag{8}$$

where k is the vapor concentration, u is the specific energy, T is the temperature, v is the velocity, R is the bubble

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Nomenclature

a	thermal diffusivity
R	bubble radius
$\dot{R}$	time derivative of the radius
r	radial Euler coordinate
t	time
T	temperature
ρ	density
p	pressure
k	vapor concentration in the bubble
D	diffusion coefficient
B	gas constant
v	radial velocity
$\dot{v}$	time derivative of velocity
C	specific heat
λ	thermal conductivity
u	specific energy
w	diffusion rate
j	rate of mass transfer per unit interface surface
x	coefficient for temperature jump at the interface
X	accommodation coefficient
μ	viscosity
P_∞	hydrostatic pressure
ℓ	specific heat of vaporization
σ	surface tension coefficient
Γ	specific heat ratio of gas–vapor mixture
ω	circular frequency
α	nondimensional displacement of bubble surface
c_p	specific heat of the gas at constant pressure
c_v	specific heat of the gas at constant volume
P_A	acoustic pressure amplitude
f	$\frac{\omega}{2\pi}$ frequency of oscillations
θ	nondimensional small deviation of the temperature
K	nondimensional small deviation of the concentration
$\underline{P}$	nondimensional small deviation of the pressure
τ	$\frac{t}{t_D}, t_D = \frac{R_0^2}{D}$
J	$j/j_0, j_0 = \frac{Dp_0}{3R_0}$
H	$\mathrm{i}\omega R_0^2/D,\ i = \sqrt{-1}$
V	$\frac{vR_0}{D}$ nondimensional velocity
ξ	$\frac{r}{R_0}$ nondimensional radial coordinate
S	$\frac{2\sigma}{R_0 p_0}$
M	$\frac{B_0 T_0}{l}$
Le_0	$\frac{a_0}{D}$ Lewis number
Pe	$R_0(p_0/\rho_\mathrm{e})^{1/2}/D$ Peclet number
Ω	$\omega R_0^2/D$

Subscripts

l	liquid
v	vapor
g	gas
s	at saturation
σ	at bubble surface
o	in equilibrium
∞	conditions in infinity

radius, D is the interdiffusion coefficient, c_p and c_v are the specific heats of the mixture at constant pressure and volume, c_ℓ is the specific heat of the liquid, λ is the thermal conductivity, w is the diffusion rate, $v_{\mathrm{e}\sigma}$ is the particle velocity of the liquid at the bubble surface, and B is the gas constant.

In the event of a nonequilibrium phase transition, the temperature at the phase interface suffers a discontinuity, the magnitude of which is given by the relation [10].

$$[T] = T_{\mathrm{e}\sigma} - T_{\mathrm{v}\sigma} = xjT_\mathrm{v}/\rho_\mathrm{v}(R)\sqrt{B_\mathrm{v}T_\mathrm{v}}. \tag{9}$$

Here j is the phase-transition rate referred to unit surface, and the subscripts o and s indicate the parameters at the surface of the bubble and on the saturation line. A molecular-kinetic analysis [10] of the structure of the temperature discontinuity at the phase interface indicates that $x = 0.32$.

For water in normal conditions this jump of temperature us negligible.

The phase-transition kinetics are described by the Hertz–Knudsen equation [11]:

$$j = \frac{\chi}{\sqrt{2\pi B_\mathrm{v}}}\left[\frac{p_\mathrm{v}(T_{\mathrm{e}\sigma})}{\sqrt{T_{\mathrm{e}\sigma}}} - \frac{p_{\mathrm{v}\sigma}}{\sqrt{T_{\mathrm{v}\sigma}}}\right]. \tag{10}$$

The standard value of the accommodation coefficient for water is $\chi = 0.04$.

The pulsations of the bubble in a viscous incompressible liquid in the presence of phase transition are described by the equation [1]:

$$R\dot{v}_{\ell\sigma} + \frac{3}{2}v_{\ell\sigma}^2 + \frac{2jv_{\ell\sigma}}{\rho_\ell} = \frac{p - p_{(\infty)} - 2\sigma/R}{\rho_\ell} - 4\frac{\mu_\ell}{\rho_\ell}\frac{v_\ell\sigma}{R}, \tag{11}$$

in which $p_{(\infty)}$ is the pressure of the liquid far from the bubble and σ is the coefficient of surface tension.

The boundary conditions are specified at the moving boundary $r = R(t)$:

$$\begin{aligned} &T_\ell = T_{\ell\sigma}, \quad T_\mathrm{v} = T_{\mathrm{v}\sigma}, \quad \lambda_\ell\frac{\partial T_\ell}{\partial r} - \lambda\frac{\partial T}{\partial T} = jl, \\ &\rho_\mathrm{v}(\dot{R} - v - w_\mathrm{v}) = \rho_\ell(\dot{R} - v_\ell) = j, \\ &\rho_g(\dot{R} - v - w_\mathrm{g}) = 0. \end{aligned} \tag{12}$$

Here ℓ is the specific heat of vaporization. Also,

$$\frac{\partial k}{\partial r} = \frac{\partial T}{\partial r} = 0 \text{ at } r = 0, \quad T_\ell = T_0 \text{ at } r = \infty. \tag{13}$$

If the pressure uniformity condition holds, the energy equation for the gas phase has the integral

$$\frac{\mathrm{d}p}{\mathrm{d}t}=\frac{3}{R}\left[-\Gamma p v_{\sigma}+\frac{p(B_{\mathrm{v}}-B_{\mathrm{g}})}{B}D\frac{\partial k}{\partial r}|_{R}+(\Gamma-1)\lambda\frac{\partial T}{\partial r}|_{R}\right], \tag{14}$$

where $\Gamma = c_{p/}c_{\mathrm{v}}$ is the specific heat ratio for the gas–vapor mixture, v_{σ} is the mass velocity of the vapor–gas mixture at the bubble surface. The Clausius–Clapeyron equation far from critical state, when $\rho_{\mathrm{v}} \ll \rho_{\mathrm{e}}$, can be written in the form

$$\frac{\mathrm{d}p_{\mathrm{v}}}{\mathrm{d}T}=\frac{\ell\rho_{\mathrm{v}}}{T}. \tag{15}$$

Let us consider the small oscillations of bubble under the action of a sound pressure

$$p_{(\infty)} = p_{\infty} + p_{\mathrm{A}}\mathrm{e}^{\mathrm{i}\omega t}, \quad p_{\mathrm{A}} \ll p_{\infty}, \tag{16}$$

where p_{∞} is hydrostatic pressure, ω is the frequency.

In this case the bubble radius can be described by the real part of the expression:

$$R = R_0[1+\alpha\exp(\mathrm{i}\omega t)], \quad |\alpha| \ll 1. \tag{17}$$

The system of fundamental Eqs. (1)–(17) is linearized. Let P, θ, and K be small deviations of the pressure, temperature, and concentration from the equilibrium state:

$$\begin{aligned} &p = p_0[1+P(\tau)], \quad T = T_0[1+\theta(r,\tau)], \\ &k = k_0[1+K(r,\tau)]. \end{aligned} \tag{18}$$

We assume that

$$\begin{aligned} &P = P^0\mathrm{e}^{H\tau}, \quad \theta = \theta^0(r)\mathrm{e}^{H\tau}, \quad K = K^0(r)\mathrm{e}^{H\tau}, \\ &J = J^0\mathrm{e}^{H\tau}, \quad V = V^0\mathrm{e}^{H\tau}, \\ &V = vR_0/D, \quad \tau = t/t_D, \quad H = \mathrm{i}\omega R_0^2/D, \\ &J = j/j_0, \quad t_D = R_0^2/D, \quad j_0 = D\rho_0/3R_0. \end{aligned} \tag{19}$$

After linearization and transformation to dimensionless variables, making use of relations (17)–(19) and the condition $\rho_{\mathrm{v}} \ll \rho_{\ell}$ we rewrite the system of fundamental equations in the form

$$V_{\sigma} = \alpha H - J/3, \quad V_{\ell\sigma} = \alpha H, \tag{20}$$

$$P = HV_{\ell\sigma}/Pe^2 - S\alpha, \tag{21}$$

$$J = 3\chi k_1 k_0\left[\theta_{\ell\sigma} - M\left(P + \frac{B_{\mathrm{g}}}{B_0}K|_{\xi=1}\right)\right], \tag{22}$$

$$HP = 3\Gamma\left[Le_0\frac{\partial\theta}{\partial\xi}|_{\xi=1} - V_{\sigma} + k_0k_2\frac{\partial K}{\partial\xi}|_{\xi=1}\right], \tag{23}$$

$$H\theta = Le_0\nabla^2\theta + (1-1/\Gamma)HP - k_0k_3HK, \tag{24}$$

$$HK = \nabla^2 K, \tag{25}$$

$$H\theta_{\ell} = Le_{\ell}\nabla^2\theta_{\ell}, \tag{26}$$

$$\frac{\partial K}{\partial\xi}|_{\xi=1} = J(1-k_0)/3k_{0,} \tag{27}$$

$$\begin{aligned} &\theta_{\ell\sigma} - \theta_{\mathrm{v}\sigma} = k_4J/k_0, \\ &\xi = r/R_0, \quad \nabla^2\theta = \theta_{\xi\xi} + 2\theta_{\xi}/\xi, \\ &S = 2\sigma/R_0p_0, \\ &M = B_0T_0/l, \\ &k_1 = R_0l/D\sqrt{2\pi B_{\mathrm{v}}T_0}, \\ &k_2 = (B_{\mathrm{v}}-B_{\mathrm{g}})/B_0\Gamma_{,} \quad k_3 = (B_{\mathrm{v}}-B_{\mathrm{g}})/c_p, \\ &k_4 = xD/3R_0\sqrt{B_{\mathrm{v}}T_0}, \end{aligned} \tag{28}$$

$$\begin{aligned} &Le_0 = a_0/D, \quad Le_{\ell} = a_{\ell}/D, \quad a_{\ell} = \lambda_{\ell}/\rho_{\ell}c_{\ell}, \\ &a_0 = \lambda_0/\rho_0c_p, \end{aligned} \tag{29}$$

$Pe = R_0(p_0/\rho_{\ell})^{1/2}/D$ is the diffusion Peclet number.

Solutions of Eqs. (24)–(26) satisfying the boundary conditions at $r = R_0$ and $r \to \infty$ as well as the condition of finiteness of the temperature and concentration at the center of the bubble can be written in the form

$$\begin{aligned} \theta = {} & A\frac{\sinh(H_2^{1/2}\xi)}{\xi} + \left(1-\frac{1}{\Gamma}\right)P \\ & + \frac{k_0k_3K|_{\xi=1}}{(1-Le_0)\xi}\left[\frac{\sinh(H_2^{1/2}\xi)}{\sinh H_2^{1/2}} - \frac{\sinh(H^{1/2}\xi)}{\sinh H^{1/2}}\right], \end{aligned} \tag{30}$$

$$\theta_{\ell} = \theta_{\ell\sigma}\exp[H_1^{1/2}(1-\xi)]/\xi, \tag{31}$$

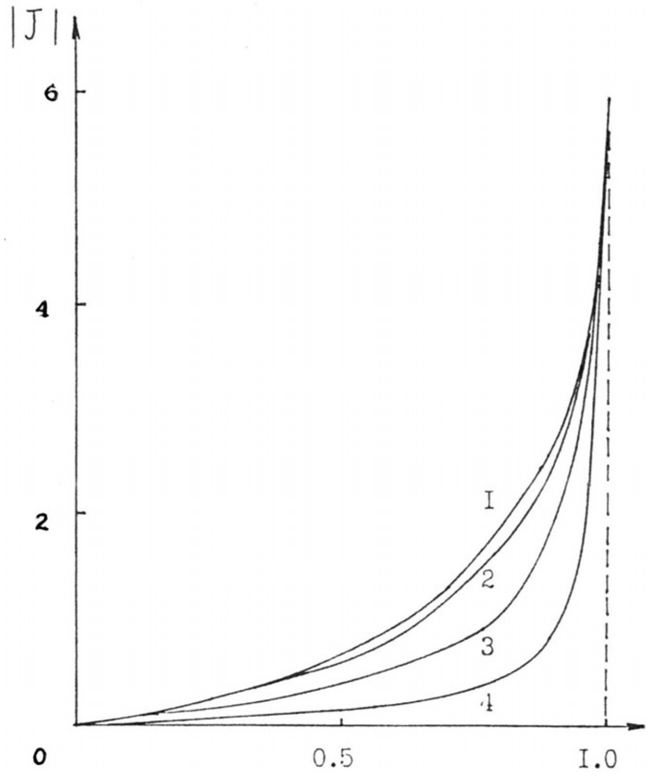

Fig. 1. Dependence of the phase-transition rate amplitude on the equilibrium vapor concentration. $R_0 = 0.1$ mm; $f = 500$ Hz. Curves 1–4 correspond to the values of diffusion coefficient $D = 2.5\times10^{-4}; 2.5\times10^{-5}; 2.5\times10^{-6}; 2.5\times10^{-7}\left(\frac{\mathrm{m}^2}{\mathrm{s}}\right)$, respectively.

$$K = A_1 J \sinh(H^{1/2}\xi)/\xi,$$

$$A = [\theta_{\ell\sigma} - k_4 J/k_0 - (1 - 1/\Gamma)P]/\sinh H_2^{1/2},$$

$$J = A_2 P, \quad \theta_{\ell\sigma} = A_3 P, \quad H_2 = H/Le_0, \quad H_1 = H/Le_\ell,$$

$$A_1 = (1 - k_0)/3k_0 B_1 \sinh H^{1/2}, \quad A_3 = M + k_6 A_2, \tag{32}$$

$$A_2 = \frac{(1 + H_1^{1/2})M\lambda_\ell/\lambda_0 + B_2(M - 1 + 1/\Gamma)}{B_2 k_4 - A_4 + k_3(1 - k_0)k_0(1 - B_2/B_1)/3(1 - Le_0)},$$

$$A_4 = k_0/k_5 + k_6[\lambda_\ell(1 + H_1^{1/2})/\lambda_0 + B_2],$$

$$k_5 = 3c_p T_0 Le_0/l,$$

$$k_6 = [1 + \chi k_1(1 - k_0)MB_{\rm g}/B_0 B_1)/3\chi k_1,$$

$$B_1 = H^{1/2}\coth H^{1/2} - 1, \quad B_2 = H_2^{1/2}\coth H_2^{1/2} - 1 \tag{33}$$

From the system of Eqs. (16)–(33) we can find analytical expression for nondimensional rate of phase transitions

$$
\begin{aligned}
&J = j/j_*, \quad j_* = \frac{p_{\rm A}}{3\rho}\sqrt{\frac{p_\infty}{\rho_\ell}},\\
&J = A_J \frac{E_2 \exp({\rm i}\omega t)}{3\Gamma p_0 - (\rho_\ell \omega^2 R_0^2 + 2\sigma/R_0 - 4\mu_\ell DH/R_0^2)E},\\
&E_1 = \frac{k_0(1 + k_2(1 - k_0))}{H} + \frac{3B_2(k_6 - k_4)}{H_2} + \frac{k_3(1 - k_0)k_0}{H_2(1 - Le_0)}\left(\frac{B_2}{B_1} - 1\right),\\
&E_2 = \frac{(1 + H_1^{0.5})M\lambda_\ell/\lambda_0 + B_2(M - 1 + 1/\Gamma)}{k_3(1 - k_0)k_0(1 - B_2/B_1)/3(1 - Le_0) + B_2(k_4 - k_6) - k_0/k_5 - k_6(1 + H_1^{0.5})\lambda_\ell/\lambda_0},\\
&E = 1 - 3\Gamma B_2(M - 1 + 1/\Gamma)/H_2 - \Gamma E_1 E_2,\\
&A_J = 3\Gamma k_0 D\rho_0 \ell(\rho_{\rm e}/p_\infty)^{1/2}/R_0,
\end{aligned}
\tag{34}
$$

Calculations were made according to (34) for the bubbles containing steam and air, oscillating in water. Hydrostatic pressure was 0.1 MPa.

Fig. 1 presents the dependence of phase-transition amplitude as function of equilibrium vapor concentration. $R_0 = 0.1$ mm; $f = 500$ Hz ($f = \omega/2\pi$). Curves 1–4 correspond the values of diffusion coefficient:

$D = 2.5 \times 10^{-4}; 2.5 \times 10^{-5}; 2.5 \times 10^{-6}; 2.5 \times 10^{-7} (\frac{\rm m^2}{\rm s})$, respectively. One can see that the diffusional retardation of phase transition rate takes place. Dependence is strictly nonlinear. When $k_0 \leqslant 0.5$ the properties of the vapor–gas bubble are similar to the properties of a gas bubble. When $k_0 \leqslant 0.2$ the phase-transition rate is practically zero. The presence of very small amount of inert gas in the bubble reduces the phase-transition rate significantly. This effect is particularly conspicuous for small values of the diffusion coefficient. This is attributable to the fact that the phase-transition rate decreases with a decrease in D, since the vapor component loses its ability to penetrate rapidly through the shielding gas layer on the surface of the bubble. For the real value of $D = 2 \times 10^{-5}$ m^2/s the presence of 10% of inert gas decreases the amplitude of phase-transition rate on 50%.

Fig. 2 presents the dependence of the phase-transition amplitude as function of equilibrium vapor concentration for different value of accommodation coefficient. $R_0 = 0.1$ mm, $f = 500$ Hz. Curves 1–3 correspond to the values of $\chi = 0.4$; 0.04; 0.004, respectively. Calculations show that for $\chi \geqslant 0.04$ the phase-transition rate dependence on concentration practically coincide with limiting equilibrium case ($\chi = \infty$). Thus the effect of χ (if $\chi \geqslant 0.04$) is practically the same as in equilibrium case. For large content of inert gas in the bubble ($k_0 \leqslant 0.5$) accuracy of the value of χ is not very important. For vapor bubbles small variations of the value of accommodation coefficient in the area $\chi \leqslant 0.04$ leads to significant change of phase-transition rate. For these cases the correct choice of this parameter is necessary.

Fig. 3 shows the effect of the parameter $\Omega = \omega R_0^2/D$ on the dependence of phase transition rate as function of the equilibrium vapor concentration. $R_0 = 10^{-5}$ m; curves 1–4 correspond to the values of $f = 10^3; 10^4; 10^5; 10^6$ Hz. In [4] for every value of R_0 it was shown the existence of critical vapor concentration k^* at which the amplitude of the bubble oscillations grows without bound. Because of the linearity of the problem this critical concentration k^* will be also critical for the amplitude of phase-transition rate.

The resonance effect is decreasing with the growth the value of Ω.

This decrease takes place even for $\Omega \geqslant 10$. The further growth of Ω leads to the decrease of phase-transition rate up to zero. The dependence of the amplitude of phase-transition rate as function of concentration became monotonic for large value of Ω. This follows also from formula (34) if we will calculate limit when $\Omega \to \infty$.

The experimental existence of founded effect was established earlier (see for example [12]).

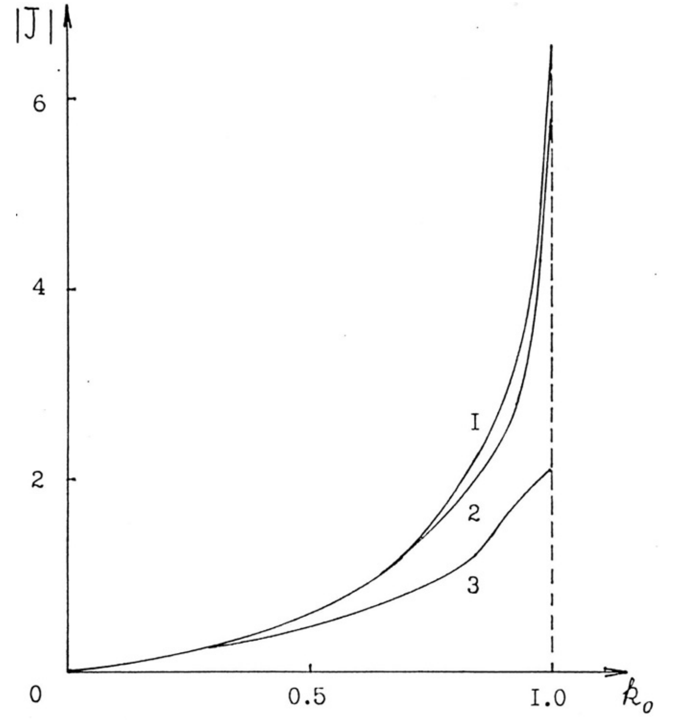

Fig. 2. The effect of accommodation coefficient on the dependence of phase-transition rate amplitude on the vapor concentration. $R_0 = 0.1$ mm; $f = 500$ Hz. Curves 1–3 correspond to the values of $\chi = 0.4; 0.04; 0.004$, respectively.

2. Conclusion

The effect of phase-transition retardation is investigated for the radial oscillations of vapor–gas bubbles in liquids. It is shown that the presence of a small amount of inert gas can lead to considerable decrease of the phase transition rate.

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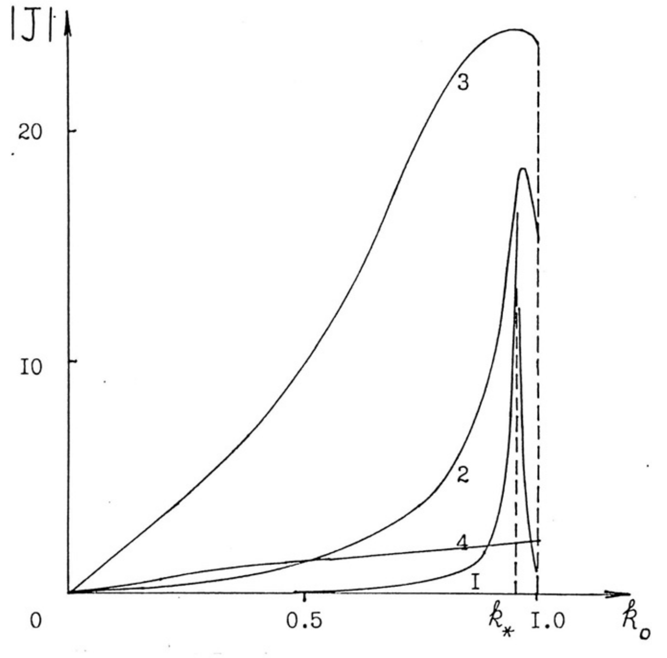

Fig. 3. The effect of the acoustic field frequency on the dependence of phase-transition rate amplitude on the vapor concentration. $R_0 = 10^{-5}$ m. Curves 1–4 correspond to the values of $f = 10^3; 10^4; 10^5; 10^6$ Hz, respectively.